Spectral statistics of disordered metals in the presence of several Aharonov-Bohm fluxes

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The form factor for spectral correlations in a diffusive metal is calculated in the presence of several Aharonov-Bohm fluxes. When the fluxes ϕ are equal, the correlations are universal functions of $ng^2\phi$ where g is the dimensionless conductance and n is the number of applied fluxes. This explains recent flux dependence of the correlations found numerically at the metal-insulator transition.

In the recent past, there has been several studies on the effect of a Aharonov-Bohm flux on the spectral properties of disordered metals. These works were mainly stimulated by the observation of persistent currents in mesoscopic rings [1]. In addition, the flux can be used as a tool to probe the evolution of the statistical properties of spectra between different symmetry classes [2,3,4,5]. When no flux is applied, time reversal symmetry is preserved, the low energy spectral correlations are those of an ensemble of real random matrices [6,7,8]. This ensemble is invariant under orthogonal transformations and is called the Gaussian Orthogonal Ensemble (GOE). In a finite flux, time reversal symmetry is broken and the relevant ensemble of random matrices is the Gaussian Unitary Ensemble (GUE). One or several magnetic fluxes drive the transition between the two ensembles. A magnetic flux is also an example of external parameter which can probe the parametric correlations, i.e. correlations of energy levels between different values of a parameter [9].

Quite recently, a Aharonov -Bohm flux has been applied in numerical experiments to study the transition between universality classes at the metal-insulator transition [10] . In this work, several fluxes were applied in perpendicular directions and the authors noticed that the cross over depends on the number of applied fluxes.

The aim of this short note is to describe analytically the effect of several fluxes. Namely, I extend the semiclassical calculation of the spectral correlations in metals, and in particular of the form factor, to the case where several fluxes are applied. Consider a three-dimensional disordered metal in the diffusive regime, i.e. when all the dimensions L_x, L_y and L_z are larger than the mean free path l_e, so that the electronic motion is diffusive in the three directions. Periodic boundary conditions are imposed in the three directions but the hypertorus can be pierced by magnetic fluxes ϕ_x, ϕ_y, ϕ_z in three perpendicular directions. It is known that the effect of a flux line is simply to change the boundary conditions so that:

$$\psi(x+L_x,y,z) = e^{i2\pi\varphi_x}\psi(x,y,z) \quad (1)$$

$$\psi(x,y+L_y,z) = e^{i2\pi\varphi_y}\psi(x,y,z) \quad (2)$$

$$\psi(x,y,z+L_z) = e^{i2\pi\varphi_z}\psi(x,y,z) \quad (3)$$

where $\varphi_i = \phi_i/\phi_0$, with $i = x, y, z$. ϕ_0 is the flux quantum.

To describe the spectral correlations, I use the semiclassical result found by Argaman *et al.* [11] to describe the two-point correlation of the density of states $K(\epsilon) = \langle\rho(E)\rho(E+\epsilon)\rangle - \langle\rho(E)\rangle^2$. These authors have related the form factor $\tilde{K}(t)$, the Fourier transform of this two-point correlation function ($\hbar = 1$ throughout the paper):

$$\tilde{K}(t) = \frac{1}{2\pi}\int K(\epsilon)\exp(i\epsilon t)dt \quad (4)$$

to the return probability $P(t)$ for a diffusive particle. In the presence of magnetic fluxes φ_i, this relation is:

$$\tilde{K}(t,\varphi_i) = tP(t,\varphi_i)/(4\pi^2) \quad . \quad (5)$$

When the phase of electrons is coherent, the return probability contains two terms, the classical return probability P_{cl} and an interference term, P_{int}, which results from phase coherence between a diffusive path and the same path obtained by time reversal symmetry. The latter term depends on the magnetic fluxes. It is the solution of the diffusion equation and is given by:

$$P_{int}(t,\varphi_i) = \sum_{\{q_x,q_y,q_z\}} e^{-D(q_x^2+q_y^2+q_z^2)t} \quad (6)$$

where the wave vectors of the diffusion modes are determined by the boundary conditions: $q_i = 2(n_i + 2\varphi_i)\pi/L_i$ and $n_i = 0, \pm1, \pm2, \cdots$. D is the diffusion coefficient. For our purpose, it is more convenient to rewrite this interference term as a sum over winding numbers around each flux tube:

$$P_{int}(t,\varphi_i) = \frac{L_xL_yL_z}{(4\pi Dt)^{3/2}} \sum_{\{m_i\}=-\infty}^{\infty} e^{-\sum_i (m_i^2L_i^2)/4Dt} \times \cos(4\pi\sum_i m_i\varphi_i) \quad (7)$$

L_i the dimension of the system in the direction i. The sums are over $i = x, y, z$. This expression can be deduced from eq. 6 by Poisson summation. Quite simply, this result expresses that the m_i^{th} harmonic of the flux dependent part of the return probability depends on the probability to return to the origin after m_i windings on the hypertorus. This is a simple generalization of the one flux case [12]. Thus the spectral correlations, in particular the form factor $\tilde{K}(t)$, are related to the way the phase is accumulated along paths of diffusive trajectories after enclosing the different fluxes.

In the limit of small fluxes $\varphi_i \ll 1$, the flux periodicity can be neglected and the above sums can be replaced by gaussian integrals. One gets:

$$P_{int}(t, \varphi_i) = \exp[-16\pi^2 \sum_i E_{ci} \varphi_i^2 t]$$

where $E_{ci} = \hbar D / L_i^2$ is the Thouless energy in the direction i. The form factor is thus given by:

$$\tilde{K}(t, \varphi_i) = \frac{t}{4\pi^2}[1 + e^{-16\pi^2 \sum_i E_{ci}\varphi_i^2 t}]$$

The first term of this expression comes from the classical return probability which is equal to the interference term when the fluxes are zero.

When all the dimensions and applied fluxes are identical, $\phi_i = \phi$, $E_{ci} = E_c$, a situation considered in ref. [10], the form factor depends on the single combination of parameters $nE_c\varphi^2 t$. This implies that the flux dependence of the two-point correlation function is solely governed by $ng\varphi^2$ where $g = E_c/\Delta$ is the dimensionless conductance, n is the number of applied fluxes and Δ is the mean level spacing. As expected, the transition between orthogonal and unitary symmetries is thus faster when three fluxes are applied.

Our calculation has been done in the semiclassical regime where the time is smaller than the Heisenberg time $\tau_H = h/\Delta$. The form factor for the orthogonal-unitary transition is completely known from Random Matrix theory [3] or from the non-linear σ model [5]. More generally, one expects all correlations functions to be universal functions of the combination $ng\varphi^2$.

Let us now come to the spectral correlation at the metal-insulator transition. At the transition, the conductance g is scale independent so that the transition between orthogonal and unitary symmetry classes is expected to be size independent. This scale invariance has been found numerically recently by Batsch *et al.* [10]. In their work, they study the evolution of the distribution $P(s)$ of spacings between nearest energy levels. They introduce the parameter $\Gamma_{max}(\varphi) = (P_m(\varphi) - P_m^o)/(P_m^u - P_m^o)$, where P_m is the maximum of the spacing distribution for the orthogonal (P_m^o), unitary (P_m^u) and intermediate ($P_m(\varphi)$) cases. To describe the cross-over between orthogonal and unitary symmetries, they present different plots of the variation $\Gamma_{max}(\varphi)$, for the three cases where one, two or three identical fluxes are applied. They find that each flux dependence is scale invariant at the transition, in accordance with the scale invariance of the conductance. However, they obtain three different variations (fig.3 in ref. [10]), depending on the number of applied fluxes, a result that they qualify as "unexpected behavior".

As for $K(\epsilon, \varphi)$, one expects the quantity $\Gamma_{max}(\varphi)$ to be a universal function of $ng\varphi^2$ in the metallic regime. One may now wonder if this scaling also holds at the metal-insulator transition. If it is so, one expects that the three curves obtained in fig.3 of ref. [10], for one, two and three fluxes, merge into a *single universal curve* by rescaling $\varphi \to \varphi/\sqrt{n}$. This is indeed the case as it is shown on fig.1. I have used the data of the fig.3 in ref.[[10]]. The flux scales have been renormalized as $\varphi \to \varphi\sqrt{3}$ for one flux, $\varphi \to \varphi\sqrt{2}/\sqrt{3}$ for two fluxes, so that all the curves now coincide with the three-fluxes curve. This shows that indeed $\Gamma_{max}(\varphi)$ is a unique function of $n\varphi^2$.

It may appear surprising that the scaling found in the diffusive regime still applies at the Anderson transition. At the transition, the correlation functions have indeed a different structure. In particular, it has been found recently that the form factor is a function of $\varphi^2 t^{1+\eta/d}$ where η is an exponent related to the multifractal structure of the wave function at the transition [13]. However, several fluxes still contribute additively to the total accumulated phase so that the orthogonal-unitary cross-over is still a unique function of $ng^*\varphi^2$ at the transition, where g^* is scale invariant.

In conclusion, we have calculated the spectral form factor in the presence of several Aharonov-Bohm fluxes. When these flux are equal, the Orthogonal-Unitary transition is driven by the combination of parameters $ng\varphi^2$. This result still holds at the transition and explains recent numerical results.

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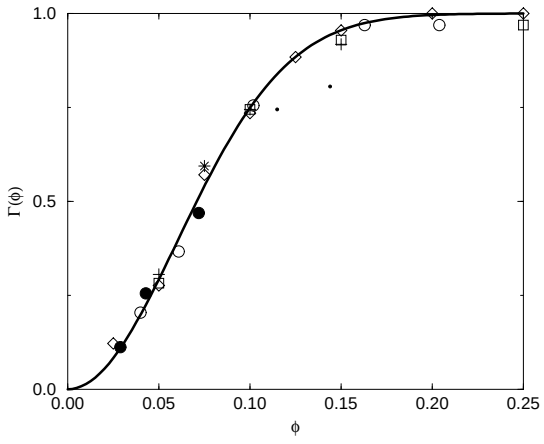

FIG. 1. The universal function $\Gamma_{max}(\varphi)$ after the rescaling proposed in the text. The two small dots, which correspond to the case of one flux are outside the universal curve, because the transition cannot be completed before the flux periodicity. For the legend of the symbols, see ref. [10]. The continuous line is a fit to the data.

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